REVIEWS

Aerodynamics of Wings and Bodies. By H. ASHLEY and M. T. LANDAHL. Addison–Wesley, 1965. 279 pp. £4. 16*s*.

This book, which originates from the notes for a two-term course for graduate aeronautical engineers at M.I.T., is mainly an account of the linearized theories of subsonic and supersonic flow. There is a supplementary chapter on viscous effects, which is largely devoted to the flat-plate boundary layer, and one on the transonic small disturbance theory. This is familiar territory, and a new work must be measured against the existing treatments. Subject matter new to textbook exposition includes a treatment of non-planar lifting surfaces, where the authors have themselves contributed much of the material; a clear account of Hayes's expression for the pressure drag of a singularity distribution in supersonic flow, leading to the rich concept of supersonic area rule as formulated by Lomax and Heaslet; and a presentation of the elegant applications by Jones and Graham of the reversed flow theorems, leading to results on minimum drag configurations in linearized supersonic flow. The authors claim two novel features: they use matched asymptotic expansions as 'a unifying framework for introducing the boundary-value problems of external flow over thin wings and bodies' and they 'embrace the important role of the high-speed computer in aerodynamics'.

The appearance a year earlier in Van Dyke's *Perturbation Methods in Fluid Mechanics* has lowered the stakes for which the authors play. Nobody is likely to be consulting the present work in order to gain either an understanding of matched asymptotic expansions or a facility in their use. On the other hand, Van Dyke can now be quoted as the authority for the 'matching principles' which he seemed to be pulling from the air. The widespread use of the technique must now be judged in context rather than for its intrinsic interest. The reviewer suspects that the resulting rigour is more apparent than real and that the unity which has been achieved through the mathematical formalism is superficial, possibly concealing the physico-mathematical concepts on which the engineer must rely.

The appearance in a textbook of methods specifically devised or adapted for automatic digital computation is to be welcomed without reservation. The authors describe Watkins's method (which is very similar to Multhopp's) for calculating the lift distribution on a wing in subsonic flow and the 'Mach box' methods for the same problem in supersonic flow. In each case the extension to non-planar systems is explained. The descriptions touch on the numerical analysis but do not, for instance, relate accuracy to mesh size. It is stated, without proof, that Watkins's and Multhopp's methods converge uniformly to an exact result as greater refinement is introduced. Only at the end of the account of Watkins's method does it appear that 'special steps must be taken for swept wings with pointed vertices'. The reviewer would have liked more information about the quoted methods and more examples of successful computational

techniques. In particular, wing design methods, in which the aerodynamicist can aim at a flow which avoids the grosser violations of the assumptions of linear theory and to which more exact methods are not readily applicable, seem appropriate for the conjunction of computers and linear theory. Other fields which have been exploited by computers are the evaluation of wave drag by supersonic area rule (where some egregious procedures have been resorted to and will be again in the absence of instruction) and the construction of incompressible flow fields by surface distributions of singularities (where the achievements of the team at Douglas under A. M. O. Smith might be mentioned in preference to a further account of the transformations of Joukowsky, von Mises and Kármán–Trefftz).

The third novel feature of the book is its colloquial style. Such writing by ear makes for easy reading and the vivid turn of phrase. Unfortunately it also tempts the writer to the plausible statement, which can be stimulating, even when loosely expressed or strictly meaningless, but can also be wrong and misleading. Examples of the first abound here. Of the second, the reader should be warned against the implication on page 161 that the Evvard–Krasilshchikova formula remains exact when the forward-going Machlines from the pivotal point are reflected more than once from the subsonic leading edges. Were this the case, closed form solutions for the very low aspect-ratio rectangular plate studied by Stewartson would readily be obtainable.

For the student, other omissions and errors are significant. The definition of a slender body places no limitation on its slope or curvature. Drag is analysed without the introduction of form drag. The account of drag at supersonic speeds fails to mention the connexion between wave drag and the production of entropy and defines vortex drag wrongly, reaching the correct formula by a *non sequitur*. The use of the word 'recipe' in connexion with principal-value integrals is unfortunate, reviving those culinary associations of the subject which should finally have been laid to rest by the deductive treatment of Heaslet and Lomax in volume 6 of the Princeton Series. There is no reference to this article, to Lighthill's vital contribution on higher approximations in the same volume, to Ward's monograph on linearized supersonic flow, nor to Robinson and Laurmann's volume on wing theory, each of which supplements the present brief treatment in some important respect.

In conclusion, the book represents a step forward in several directions. New and important topics are found between hard covers for the first time, computational methods make a welcome appearance and familiar material is presented in terms of a powerful and widely applicable technique. However, its casual approach makes it unsuitable as a medium for the unguided self-instruction of the student. The reviewer doubts whether the emphasis on matched asymptotic expansions is desirable; but he is happy to record the authors' report of considerable success over three years of teaching the approach and obliged to point out his own ignorance of educational problems.

J. H. B. Smith

An Introduction to the Method of Characteristics. By M. B. ABBOTT. Thames and Hudson, 1966. 243 pp. £4. 4*s*.

Characteristics, the paths along which weak discontinuities can travel, play a fundamental role in the mathematical theory of hyperbolic partial differential equations, i.e. those describing propagation processes. The qualitative features of this role, such as the fact that the solution at a field point depends on only a limited segment of the boundary data, can be understood in terms of signal propagation. Quantitatively, the partial differential equations simplify when applied along the characteristics, reducing to total differential relations along these curves when there are only two independent variables. When these relations can be integrated and the characteristics determined explicitly, the mathematical solution of the problem can be obtained. In general, these integrations cannot all be performed, but they can in a number of important instances, giving rise to some exact solutions of non-linear, as well as linear problems. Because of its conceptual simplicity and importance and its practical usefulness, the method of characteristics finds its natural place in a number of texts on fluid dynamics, particularly those dealing with compressible flow, as well as in texts on partial differential equations.

Dr Abbott's book is an elementary introduction to the use of characteristics for propagation problems in two independent variables. It is intended primarily for civil and mechanical engineers, no extensive mathematical knowledge is required and the elementary properties of matrices and determinants used are reviewed in an appendix. As well as developing some of the mathematical theory of characteristics, three specific fields of application are discussed. These are one-dimensional unsteady shallow water theory, gas dynamics, and failure and flow problems in the theory of plasticity. There is a short account of how finite difference methods can be applied to the numerical solution of propagation problems.

Unfortunately, this book fails in its appointed task for several reasons. There are too many inaccurate statements. One is warned of this in the preface where Dr Abbott apologizes to 'mathematicians for the liberties taken with their material', and generally offers 'qualified apologies for certain over-statements and over-generalizations, and for a frequent looseness of expression', suggesting that his primary purpose is to aid conceptual understanding. It is hard to see how this is achieved by stating (page 12, line 1 *et seq.*) that the characteristics of linearized equations are always systems of parallel straight lines (they are not), or (page 73, line 10 *et seq.*) '. . . the method of characteristics represents a method whereby the difficult problem of solving either a second-order linear partial differential equation, or a pair of first-order partial differential equations, is reduced to the comparatively simple problem of solving a pair of ordinary linear differential equations'. This may be so, but is not generally true. A number of other statements require qualification. The statement (page 35) that a disturbance initiated within a region of small width remains confined within regions of small width at later times is true only when Riemann invariants exist. Otherwise such a disturbance spreads throughout its range of

influence. The author tends to discuss only problems for which Riemann invariants can be found, and lists the finding of these invariants as the second step of the method in his synthesis without mentioning its possible impossibility.

In spite of catering to the least mathematically sophisticated by writing out pages of matrix manipulations at length, often unnecessary length as when he uses a 4×5 rather than a 2×2 matrix to find the characteristic directions for channel flow, the author reveals a distracting fondness for the mathematically sophisticated theory of distributions. Though this is not integrated into the work, more references to it are made according to the subject index than to all but one other topic. By contrast 'domain of dependence', a basic concept, is mentioned once and that in the penultimate chapter.

My most important criticism of the book, however, is that the treatment of most topics is too superficial and lacking in substance. Dr Abbott does not go beyond some very elementary applications to others that are well within reach and would add considerably to the book's value. The chapter on gas dynamic applications is fairly typical. It starts with a discussion of one-dimensional unsteady homentropic flow. The similarity of the analysis to that in the previous chapter on shallow water theory is unremarked, though the equations in the latter instance are precisely those for a perfect gas with $\gamma = 2$. The centred simple wave solution that results from the sudden uniform withdrawal of a piston from gas at rest is described qualitatively, but the mathematical solution is not obtained. The possibility of shock formation due to characteristics converging in other instances is mentioned, though no example illustrating this process is worked out. Instead the author proceeds to calculate the shock discontinuity relations. There is no clear discussion of the orientation of the shock wave in relation to the characteristics, nor of the fact that the shock wave is not a characteristic even though it is a line of discontinuity, and characteristics were initially introduced as lines along which some kinds of discontinuity occur. Characteristic relations are next derived for steady plane supersonic flow but no problems solved, not even the Prandtl–Mayer expansion wave though this is described. Finally, two solutions are derived though without using the method of characteristics. These are for the spherically symmetric motion due to a sphere expanding uniformly into gas at rest. The acoustic approximation and G. I. Taylor's similarity solutions are given.

This summary illustrates the paucity of material bearing on the subject at issue and the lack of depth in its treatment. It is a pity because the method of characteristics has useful applications in diverse fields, and a better, wider and fuller account could have been written.

C. HUNTER